Name of college - S.S. College, J Bad

Deptt - Mathematics

Topic - Absolute Convergence

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class - B.Sc II

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Study Material :→

Absolutely and Conditionally Convergent Series :→

The Series Σu_n which contains both the +ve and -ve Terms is said to be absolutely convergent if the series $\Sigma|u_n|$ is convergent

In other words

The series which remains convergent when all the -ve terms in it are made positive, then it is called absolutely convergent series.

Ex1 :→ $1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \cdots$

is absolutely convergent

Since the series $\Sigma|u_n| = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} \cdots = \Sigma V_n$ say

$$V_n = \frac{1}{2^{n-1}}$$

$$V_{n+1} = \frac{1}{2^n}$$

$$\lim_{n\to\infty} \frac{V_n}{V_{n+1}} = \lim_{n\to\infty} \frac{1}{2^{n-1}} \times \frac{2^n}{1} = 2 > 1$$

then By ratio Test

The series Σv_n i.e. $\Sigma |u_n|$ is convergent.

Also in Σu_n we find that terms are alternately +ve and −ve, its terms are continually decreasing and

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{2^{n-1}} = 0$$

Thus all three conditions of Leibnitz Test are satisfied and as such Σu_n is convergent.

Hence the given series Σu_n is absolutely convergent.

Ex 2. Test the absolute convergence of the series

$$1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$$

Solution :→ Let

$$\Sigma u_n = 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$$

then $\Sigma |u_n| = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots$

$$= \Sigma \frac{1}{n^{1/2}}$$

Since $\Sigma \frac{1}{n^p}$ is divergent if $p \leq 1$

and is convergent if $p >$ 1

Here $p = 1/2 < 1$ and hence Σu_n is divergent

Also in the series Σu_n we find that the terms are alternately +ve and −ve.

Also its terms are continually decreasing

and $\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

Here we see all the required conditions of Leibnitz Test are satisfied and therefore Σu_n is convergent.

Hence Σu_n is absolutely ~~conditionally~~ convergent.

Ex:→ Is the series $\sum_{n=1}^{\infty} (-1)^{n-1} \left[\frac{1}{n\sqrt{n}}\right]$ absolutely convergent.

<u>Solution</u> :→ Let $\Sigma u_n = \sum_{n=1}^{\infty} (-1)^{n-1}\left(\frac{1}{n\sqrt{n}}\right)$ —①

then $\Sigma |u_n| = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} = \Sigma v_n$ say

From ① In the series Σu_n we find that

(a) The terms are alternately +ve and −ve

(b) The terms are continually decreasing and

(c) $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{n\sqrt{n}} = 0$

Hence all the conditions of Leibnitz Test are satisfied and such as Σu_n is convergent.

Again comparing Σv_n with the Auxiliary series $\Sigma v_n = \Sigma \frac{1}{n^{3/2}}$, we find that Σv_n is convergent.

thus we find $\Sigma |u_n|$ is convergent series

Hence Σu_n is absolutely convergent.